Use sigma notation to write the series $\frac{7}{75}-\frac{14}{71}+\frac{28}{67}-\cdots+\frac{1792}{43}$. ← GEOMETRIC $r=-2$ ← ARITHMETIC $d=-4$

SCORE: _____ / 20 PTS

Show clearly the algebra used to find the upper limit of summation (ie. not just by counting).

$$\sum_{n=1}^{9} \frac{7\cdot(-2)^{n-1}}{75-4(n-1)} = \sum_{n=1}^{9} \frac{7\cdot(-2)^{n-1}}{79-4n}$$

$79-4n=43$

IF $n=9$

Find parametric equations for the ellipse with $(-1, 7)$ and $(-1, -5)$ as foci, and $(3, 1)$ as one endpoint of the minor axis.

SCORE: _____ / 20 PTS

F $\cdot (-1,7)$

C $\cdot (-1,1) \quad \cdot (3,1)$

F $\cdot (-1,-5)$

$a^2 = 6^2 + 4^2$

$a^2 = 52$

$a = 2\sqrt{13}$

$x = -1 + 4\cos t$

$y = 1 + 2\sqrt{13}\sin t$

Find the coefficient of $x^{16}y^{42}$ in the expansion of $(9x^4 - 11y^2)^{25}$. **SCORE: _____ / 20 PTS**

Your final answer may use $+, -, \times$ and positive powers. It may **NOT** use $\cdots$, !, $\div$, negative exponents **NOR** fractions.

It does **NOT** need to be simplified into a single number. **Show all work as demonstrated in lecture.**

$$\sum_{i=0}^{25} \binom{25}{i} (9x^4)^{25-i} (-11y^2)^i$$

$$= \sum_{i=0}^{25} \binom{25}{i} 9^{25-i} x^{4(25-i)} (-11)^i y^{2i}$$

$4(25-i) = 16$ AND $2i = 42$

IF $i = 21$

$$\binom{25}{21} 9^4 (-11)^{21}$$

$$= \frac{25 \cdot \cancel{24} \cdot 23 \cdot 22}{\cancel{4 \cdot 3 \cdot 2} \cdot 1} \cdot 9^4 (-11)^{21}$$

$$= 25 \cdot 23 \cdot 22 \cdot 9^4 \cdot (-11)^{21}$$

Prove by mathematical induction: $\sum_{i=1}^{n} 2i^2 = \frac{n(n+1)(2n+1)}{3}$ for all integers $n \geq 1$. **SCORE: _____ / 30 PTS**

<u>HINT: The algebra in the proof is much easier if you use factoring.</u>

$$\sum_{i=1}^{1} 2i^2 = 2(1)^2 = 2 = \frac{1(2)(3)}{3}$$

ASSUME $\sum_{i=1}^{k} 2i^2 = \frac{k(k+1)(2k+1)}{3}$ FOR SOME PARTICULAR BUT ARBITRARY INTEGER $k \geq 1$

$$\sum_{i=1}^{k+1} 2i^2 = \sum_{i=1}^{k} 2i^2 + 2(k+1)^2$$

$$= \frac{k(k+1)(2k+1)}{3} + 2(k+1)^2$$

$$= (k+1)\left[\frac{k(2k+1)}{3} + 2(k+1)\right]$$

$$= (k+1)\left[\frac{2k^2 + k + 6k + 6}{3}\right]$$

$$= \frac{(k+1)(2k^2+7k+6)}{3}$$

$$= \frac{(k+1)(k+2)(2k+3)}{3}$$

SO, BY MATHEMATICAL INDUCTION, $\sum_{i=1}^{n} 2i^2 = \frac{n(n+1)(2n+1)}{3}$ FOR ALL INTEGERS $n \geq 1$

Find rectangular equations corresponding to the parametric equations $x = \dfrac{t}{3-t}$, $\quad y = \dfrac{t+2}{t-2}$.

SCORE: _____ / 20 PTS

Write y as a function of x.

$$x(3-t) = t$$

$$3x - tx = t$$

$$3x = t + tx$$

$$3x = t(1+x)$$

$$t = \frac{3x}{1+x}$$

$$y = \frac{\frac{3x}{1+x} + 2}{\frac{3x}{1+x} - 2} \cdot \frac{1+x}{1+x}$$

$$y = \frac{3x + 2(1+x)}{3x - 2(1+x)}$$

$$y = \frac{5x+2}{x-2}$$

Consider the sequence defined recursively by $a_n = n^2 - 3a_{n-1}$, $a_1 = -1$. **SCORE: _____ / 10 PTS**

Find the first 4 terms of the sequence, and **write them as a list**.

$a_2 = 2^2 - 3a_1 = 4 + 3 = 7$

$a_3 = 3^2 - 3a_2 = 9 - 21 = -12$

$a_4 = 4^2 - 3a_3 = 16 + 36 = 52$

$-1, 7, -12, 52$

Last year was a good year for the Lee's family business. During January 2012, revenues were \$18,371 and operating costs were \$5,841.

SCORE: _____ / 30 PTS

NOTE: You must show the use of the relevant sequence and/or series formulae to earn full credit for the following problems.

[a] If revenues each month were 1.6% higher than revenues the previous month, what was the total revenue from January 2012 to August 2012 ?

$$\frac{18{,}371(1.016^8-1)}{1.016-1} = \$155{,}466.91$$

[b] If operating costs each month decreased by a fixed amount (in dollars) from the previous month, and operating costs in August 2012 were \$4,742, what were the total operating costs from January 2012 to **December** 2012 ?

$$4742 = 5841 + 7d$$

$$d = -157$$

$$\frac{12}{2}(2(5841) + 11(-157)) = \$59{,}730$$